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TREATING OF

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THOMAS WARD, Secretary and Treasurer,

Grey street, Wellington.

Editor: C. W. ADAMS, Dunedin.

THE N.Z. SURVEYOR.

NOTICES TO CORRESPONDENTS.

As the Editor cannot undertake to send out proofs for revision, it is requested that, in order to avoid mistakes, all communications be written in a legible hand, and on one side of the paper only. It would be greatly to the advantage of the Editor, printer, and all concerned, if those who cannot, or do not, write legibly would kindly have their communications copied by a type-writer before sending them for publication. As type-writers are now coming into common use, this could easily be done in most cases.

Literary matter for publication should be addressed to the Editor, Mr. C. W. Adams, Dunedin.

Business letters should be addressed to the Secretary and Treasurer, Mr. Thomas Ward, Grey street, Wellington.

The next number will be published in March, 1896, and it is requested that all communications intended for that issue should be sent to the Editor as early as possible.

TO OUR MEMBERS.

"The attention of members is particularly drawn to the decision of the Council on the matter of arrears of subscriptions which is to be brought up for confirmation at the annual meeting, to be held the first Wednesday in February, in Auckland. There is no doubt that this matter is surrounded with difficulty, as two objects have to be achieved, viz: to keep the roll of membership as large as possible, and not to offend those members who have always regularly paid their subscriptions. Members will, however, note that the matter is before them now officially, for careful consideration on their part, so that their views can be given effect to at the annual conference. Perhaps if those members who have already paid up to date be excused any payment for next year this might make the adjustment more even. Be good enough to remember that the main object of the Institute is to enroll among its ranks *all* the surveyors of the colony, as without unity nothing can be done, and a great deal yet remains to raise our profession, chiefly in the matter of remuneration of our work, in which there is much room for improvement. The idea of the Council to print yearly lists of the members for exhibition in the various Government offices and places of business throughout the colony will, I feel sure, commend itself to you. This will show the public who are the members belonging to the Institute, and so make it more widely known."

"The notice of members is directed to the annual meeting which is to be held in Auckland on the first Wednesday in February. The following nominations must be sent to the Secretary as soon as possible, so that he may have time to send out the names of those nominated, and ballot papers for members to vote upon :—

1 President.

2 Vice-Presidents.

6 Members of Council.

Yours faithfully,

THOMAS WARD.

REPORT OF COUNCIL MEETING.

December 10, 1895.—Council Meeting held. Present—the President, Vice-President, Mr. Fulton, and the Secretary.

The minutes of previous Council Meetings were confirmed.

Mr. C. H. Bridge member of Council, wrote regretting his inability to be present, and forwarding his views upon the question of arrears of subscriptions.

The Secretary reported that he had replies from the Government upon the question of erecting certain Trigs, and replacing Standard Blocks in the city of Wellington ; that the matters would be attended to when a surveyor could be spared, also, in compliance with wish of the Council, Mr. Climie was instructed to lay down a Standard at Palmerston North. Mr. Hannify, assistant on the Drainage Board, and authorised surveyor of the Colonies of Victoria and New South Wales, wrote to the Council on the matter of his obtaining a New Zealand Certificate, asking that the Council urge upon the Surveyor-General the necessity of setting up a Board of Examiners without any more delay. Mr. William Andrew, authorised surveyor, applied to be admitted as member, and forwarded a nomination paper with the names of Edward Anderson and Alfred T. Rawson as his nominees. The Council decided to inform Mr. Andrew that in accordance with the rules his nomination must be approved of by the local committee of his district.

The following accounts were passed for payment :—

	£	s.	d.
Messrs. Stone, Son, and Co., printing September Journal ...	11	6	6
Messrs. Whitcombe and Tombs	9	10	6
Secretary—half-year's salary	12	10	0
N.Z. Times, half-year	0	4	6
	£33	11	6

The account of Messrs. Whitcombe and Tombs was passed subject to the Secretary being satisfied with the charge made for printing certificate forms.

The President and the Secretary were instructed to redeposit the funds of the Institute at the Bank of New Zealand, provided that better terms were not offered by the National Bank.

Considerable discussion took place on the question of arrears of subscriptions, and it was eventually decided, upon the motion of the President, seconded by the Vice-President (1) That those members in arrears who failed to pay the money due by them be written to and informed that their names will be retained upon the list of members on the payment of the sum of two guineas (£2 2s.), the date of their entrance into the Institute to be the date of the payment of this sum ; £1 1s. of it to be considered as entrance fee, and £1 1s. as annual subscription for the first year. (2) That in future the subscription be fixed at 15s. ; 5s. to be retained by the various local secretaries for local purposes. (3) That in future all new members joining pay an entrance fee of £1 1s. and £1 1s. for the first year's subscription.

The Annual Meeting was fixed to be held at Auckland on the first Wednesday in February next, and the actual fare there and back of one delegate from each local district to be defrayed out of the funds of the Institute.

The Council also decided (4) That complete lists of all members of the Institute be printed each year for exhibition in the various government offices and places of business throughout the Colony, with addresses of each, and date of membership, and that members to be notified that if they are more than twelve months in arrears that their names will be omitted from the list.

The Council further decided that paragraphs (1), (2), (3), (4) of the minutes shall be notices of motion for the Annual Meeting next to be held.

THE APPLICATION OF PHOTOGRAPHY TO TOPOGRAPHIC SURVEYING.

By George Heimbrod.

The application of photography to surveying is based upon the accuracy with which measurements on the photographic plate may be made. Photogrammetry is the name given to that branch of photography which aims at the construction of geometrical plans from the photographic image. Its chief purpose is to measure on the plate the rectilinear co-ordinates of a point, and convert these into the corresponding angular values.

The discussion of the subject may be divided into two parts.

(1) Photogrammetric construction.

(2) Photogrammetric apparatus.

Photogrammetric Constructions.

To construct a plan from photographs, it is necessary that the point has been photographed from two positions. Its place on the plan is determined by intersection.

We have to consider two cases.

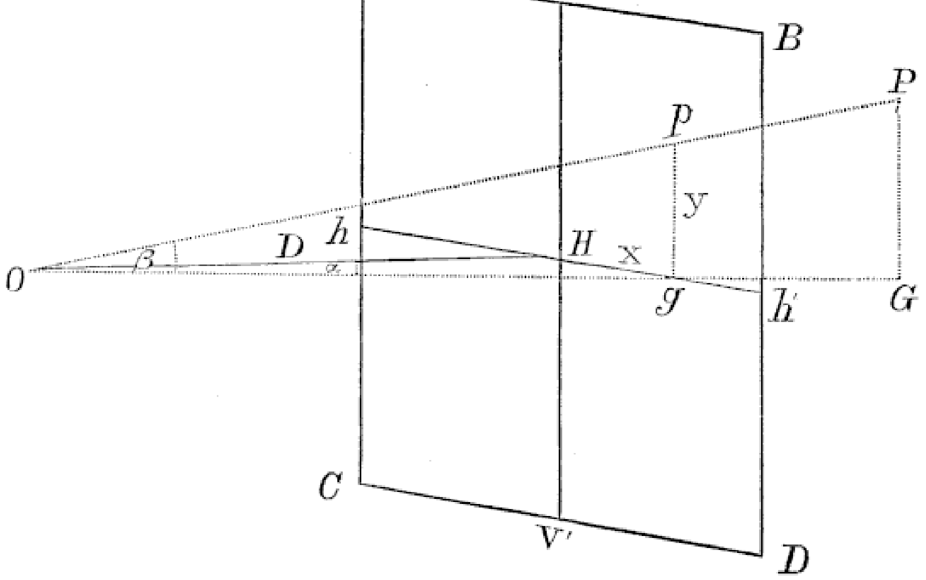
(*a*) The plate is placed vertical, or the axis of the camera is horizontal.

(*b*) The plate is inclined, in which case the axis is also inclined.

Case I.

In the following diagram A B C D represents the photographic plate, exposed when in a vertical position. H is the point in which the plate is cut by the optical axis. *h h'* is the horizontal line at right angles to the

Fig I.



optical axis, and going through H. $v\ v'$ is a line vertical to the axis, and through H.—$h\ h'$; v v' correspond therefore to the diaphragm of a theodolite. Let O be the lens, then O H is the distance of the lens from the plate (very nearly the focal distance). O H is also the optical axis. Let P be any point on the earth's surface, then its place on the plate will be at p. Its position will be fixed by the co-ordinate x, measured along h—h', and y measured along v—v'. It must be borne in mind that the position of p on the plate is reversed.

The angle α is the horizontal angle between the optical axis and the line of sight to the point p. It is found from

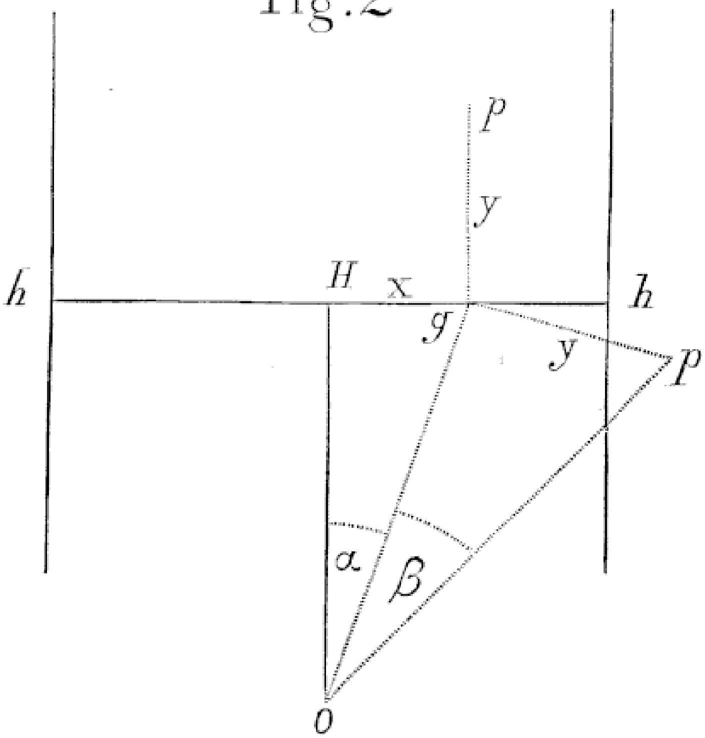
$$\tan.\ \alpha = \frac{\text{H}\ g}{\text{H O}} = \frac{x}{\text{D}}$$

β is the vertical angle, and is found from

$$\tan.\ \beta = \frac{p\ g}{\text{O}\ g} = \frac{y}{\text{D} \div \cos\ \alpha} = \frac{y}{\text{D}\ \sec\ \alpha} = \frac{y \cos\ \alpha}{\text{D}}$$

O g is found from the $\triangle$ O H g with H = 90°. α and β may also be constructed graphically. Draw $h\ h'$ (Fig. 2). Erect the perpendicular H O = D. Measure the co-ordinates on the plate and make H g = x. Join g and O, then angle H O g = α. Further erecting a perpendicular to g O = $g\ p$. Make $g\ p$ = y. Join p and O, then angle g O p = β. If the azimuth of the optical axis is known, we find by the above process the

Fig. 2



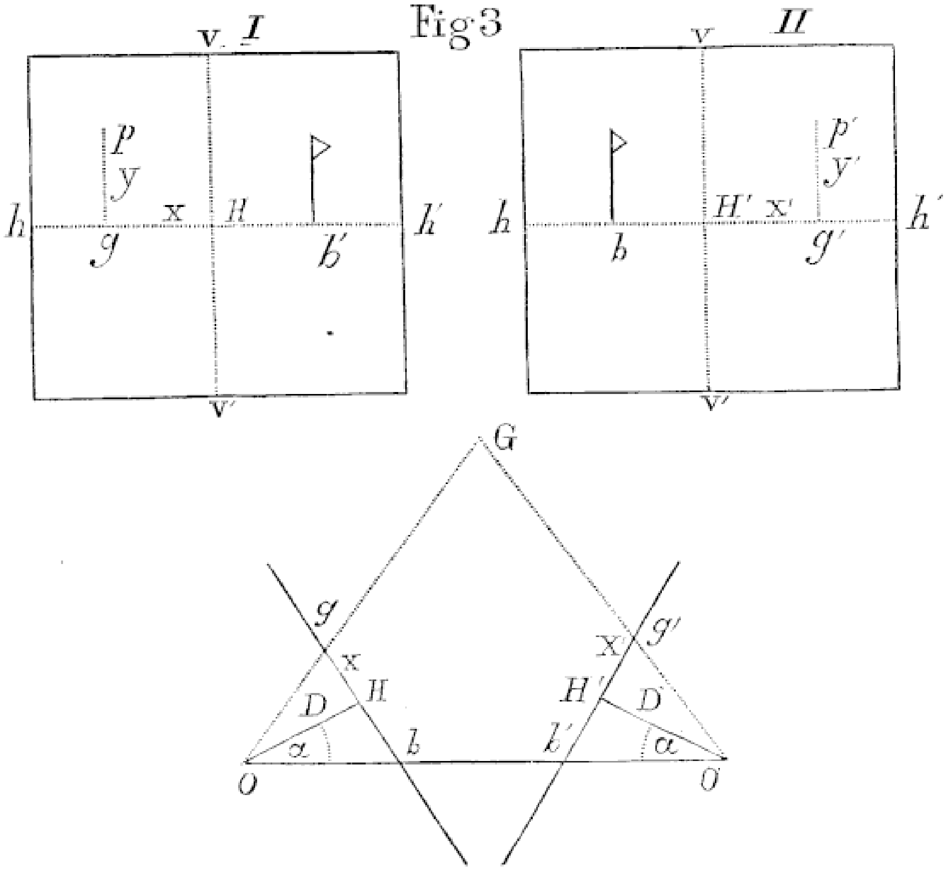
azimuths of all the other points. If now two pictures of the same part have been taken from the ends of a base, we may construct a plan by two means. Firstly, we compute for each point the angles α and β and then proceed in the usual way of laying down the points. Secondly, we may proceed graphically at once. In Fig. 3, I. and II. represent the pictures taken at the ends of a base-line. On each picture we see the flags b, b', which are the signals at the end points. Lay down the base O—O′ to any suitable scale. Compute the angle α, that is the angle which the optical axis makes with the line of sight to the signal b' at the end O′, by using the co-ordinate H′ b on plate II. Likewise compute angle α, using H b' on plate I. Lay down α, α' at O and O′. In each angle make O H = D = focal distance and O′ H′ = D′ (as a rule D = D′). Erect perpendiculars at H and H′. Now measure for each point X and y on plate I, and the corresponding x' y' on on plate II. Make H $g = x$; H′ $g' = x'$. Join O—g and O′—g', produce both lines. Their intersection G will be the position of the point with reference to the base-line. The same is done for all points. Should it happen that when a number of pictures have to be taken, one end of the base-line does not appear on any of the other plates, we simply use a station previously determined for the orientation of the new picture. This reference station must be, of course, on two consecutive plates.

The difference of elevation may be computed by taking y from the plates, also taking the values O G and O g from the plan and using the formula

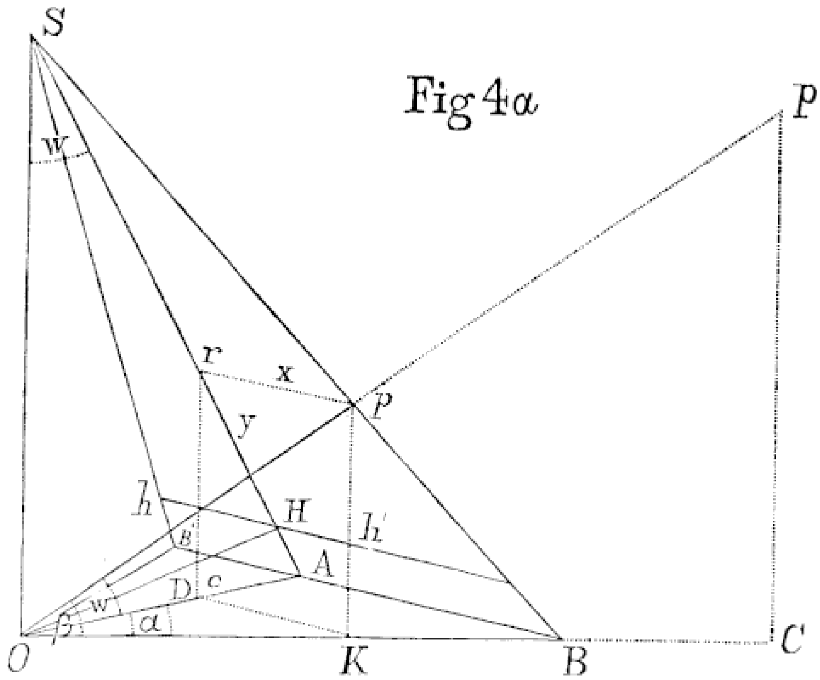
$$\triangle\ h = y\,\frac{\text{O G}}{o\ g} \qquad\qquad \triangle\ h' = y'\,\frac{\text{O}'\ \text{G}'}{o'\ g'}$$

Each point is therefore determined twice in elevation. This double determination serves as a check that the right point has been taken when measuring x, y. x', y'.

Fig 3



For the direction of the intersecting lines we have to consider only the proportions of x, x' to D and D'. These lengths may be drawn to any scale. The scale of the whole plan depends, of course, upon the scale to to which O—O' is drawn.



CASE II.

In Fig. 4a S B B' represents the plane of the picture ; B B' its intersection with the horizontal plane of O ; O H the optical axis, perpendicular to the plane of the picture ; W = inclination of O H to the horizontal. O S A is the vertical plane through the axis. All horizontal lines, being parallel to the plane of the picture, will be in the picture parallel to h h' and and B B'. All vertical lines will converge towards S, where a vertical line through O will intersect the inclined plane of the picture. A point p on the plate has the co-ordinates $p\ r$ = X, H r = y, relatively to h h' and A S. The corresponding line of sight O—P is fixed relatively to the optical axis by angle α and angle β. If we project the points r and p upon the horizon to c and K, we have c K = $r\ p$ and parallel to A B. We have further in the right-angled triangle c O K (angle c = 90°) c K = X and angle c O K = α. α is found from

$$\tan.\ \alpha = \frac{x}{O\ c}$$

Further p K parallel and $c\ r$, and the $\triangle$ p O K (K = 90°) angle p O K = β, which is found from

$$\tan.\ \beta = \frac{p\ K}{O\ K} = \frac{r\ c}{O\ K}$$

To construct these two angles graphically proceed as follows :—

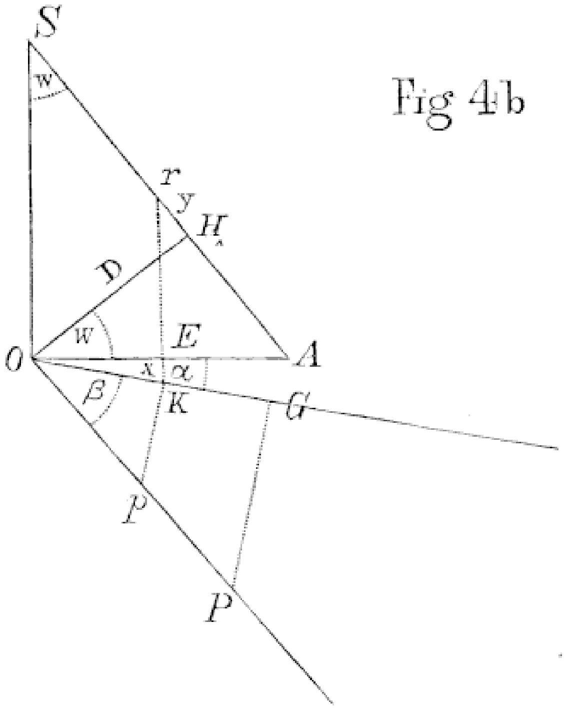


Fig 4b

Draw triangle A O S (O = 90°) and O H = D. Make H r = y. From r let fall a perpendicular to O A, which intersects O A in c. Prolong the perpendicular till c K = x. Then angle c O K = α. Erect in K on O K a perpendicular and make K p = r c, then angle p O K = β.

To construct a plan we may use again the computing or graphic method. α and β are computed from x, y, D and W = the inclination of the optical axis.

$$\tan. \alpha = \frac{x}{\text{D} \cos \text{W} - y \sin \text{W}} \qquad (1)$$

$$\tan. \beta = \frac{\text{D} \sin \text{W} + y \cos \text{W}}{(\text{D} \cos \text{W} - y \sin \text{W}) \div \cos \alpha} \qquad (2)$$

To adapt for logarithmic computations, put

$$\frac{y}{\text{D}} = \tan. \text{ M, then}$$

$$\tan. \alpha = \frac{x \cos \text{M}}{\text{D} \cos (\text{W} + \text{M})} \qquad (3)$$

$$\tan. \beta = \tan. (\text{W} + \text{M}) \cos \alpha \qquad (4)$$

or put

$$\sin \text{W} = a \qquad \text{D} \sin \text{W} = m$$
$$\cos \text{W} = b \qquad \text{D} \cos \text{W} = n$$

then

$$\tan. \alpha = \frac{x}{n - a\ y} \qquad (5)$$

$$\tan. \beta = \frac{m + b\,y}{n - a\,y} \quad \cos \alpha \qquad (6)$$

The products $a\,y$, $b\,y$ may easily be computed with a slide rule.

If we use the graphic method, we must deduce from the measured co-ordinates, x' and y', which we would have obtained had the picture been taken on a vertical plate. This is done as follows :—

Put D cot W $= a$
D cosec W $= b$, then

$$x' = \frac{a\,x}{a - y}$$

$$y' = \frac{b\,y}{a - y}$$

Having deduced x', y' from x and y for each point, we compute the orientation angles α, α' and then construct the plan by Fig. 3.

PHOTOGRAMMETRIC APPARATUS.—The pictures for photogrammetric constructions may be taken either with a specially-designed apparatus—the photo-theodolite—or an ordinary camera. In the latter case, however, several reference angles, both horizontal and vertical, must be taken, in order to determine the instrumental constants.

LENSES.—Not every lens is suitable for our purpose. The chief requirements of a lens for photogrammetry is that it shall draw in true perspective. The lenses which almost fulfil that requirement are the so-called symmetric combinations, and amongst these are, foremost, the Pantascope by Busch, Steinheil's Aplanats, and Voigtläander's Periscopes and Euryscopes. The Periscopes are constructed with a chemical focus—that is, the chemical and visual rays do not unite in the same point ; but as cameras for photogrammetric work usually work with constant focus, this is of no consequence. As only certain lenses are adapted, it will be necessary to examine each lens before using it for photo-surveying, and to determine the limit of the field in which the lens draws in true perspective.

The angular aperture of a lens depends upon the sizes of the plates and the focal distance. The larger the plates and the shorter the focal distance, the larger will be the angle. It is, however, not advisable to employ a large angle, for the image will be distorted near the edge of the lens. Dr. Koppe, of the Royal Technical College, Brunswick, used a lens with 40° aperture and plates 10 centimeters square (4 inches). The focal distance was 13·7 centimeters = ($5\frac{1}{4}$ inches). With this lens he surveyed part of the Harz Mountains, Germany.

The method for testing the lenses will be given further on.

The principal apparatus, specially designed for photo-surveying, is the photo-theodolite. It consists of an ordinary theodolite, with ex-centric telescope. The axis carrying the telescope is turned to a ring between the standards. The camera is pushed into this ring, and is kept in its place by four springs. When the camera is in its place its optical axis is parallel to the line of sight of the telescope.

INSTRUMENTAL CONSTANTS.—The constants to be determined are the chief vertical and horizontal lines, which correspond to the wires in a theodolite.

To find the vertical line with a photo-theodolite: Perform first the usual adjustments for the theodolite; then direct the telescope to a distant, well-defined mark. Move the camera up and down and mark the points at which the mark will appear on the upper and lower frame of the ground glass. A line joining these two points will be the chief vertical line. The vertical plane, which intersects the plate in the vertical, will go through the centre of the objective, and being parallel to the same plane in the telescope, will also be at right angles to the axis carrying both camera and telescope. Further, it must be examined if this plane is also perpendicular to the plate —that is, if the plate is parallel to the horizontal axis. Mark the vertical by joining the two marks on the ground glass frame by a fine wire or string. Direct this to the flame of a candle. The vertical plane of the optical axis of the camera will then go through the flame and its image on the plate. In the same plane bring the line of sight of a second theodolite by sighting to the vertical wire of the camera, then removing the camera, and seeing if the vertical wire of the theodolite will bisect the flame. If this is not the case, the theodolite will have to be shifted laterally till the adjustment is perfect. The photographic plate should now be perpendicular to the line of sight of the second theodolite. To test this, insert a plate into the camera, hang a plumb line in front of the theodolite, so that it will be covered by the vertical wire of the telescope; next observe the image of the plumb-line reflected from the plate. If everything is in adjustment, the vertical wire of the telescope, the plumb-line and its image will be in the same vertical plane. If this is not the case, turn the horizontal circle of the photo-theodolite till the adjustment is effected. The difference between this reading and the previous position will be the error of the plate. In practice it is best to shift the frame against which the plate lies during exposure. This adjustment should be repeated until no difference of circle reading appears.

The following example will illustrate this:

Circle reading in normal position = 357° 20′.0
,, ,, when plate was in
the camera = 357° 19′.5

This shows an error of $\frac{1}{2}'$ which is quite inappreciable. With plates 20 centimeters long an error of 5′ would produce an error of 0·15 millimeters on the plate, also quite inappreciable. As the plates are seldom parallel, it will be necessary to make several observations, turning the plates through 180°.

The second constant which has to be determined is the chief horizontal line, the line from which the co-ordinates for the vertical angles are measured. To find this line, we first have to make the optical axis of the camera horizontal, or the frame against which the plates will lie, vertical. Set up at some distance from the photo-theodolite a level and a levelling staff. Turn the photo-theodolite till the image of the staff on the plate is visible in the telescope of the level. If the reading on the image is the same as on the staff direct, the plate will be perpendicular to the line of sight of the level, and therefore vertical. If this is not the case, move the camera up or down till the readings coincide, and note the inclination given to the plate on the vertical circle.

Should an exposure be made with camera axis horizontal, the vertical circle must be set to this reading. If the observation is to be made with axis inclined, this reading must be added or subtracted to the inclination of the camera in order to obtain its true value. To determine now the horizontal line, make the frame vertical by the above operation, place a levelling staff in front of the camera and a level behind it. Observe the staff through the objective of the camera, and also after removing the camera. Make the readings to be the same in both cases by shifting the level in height. When this is done, mark on the frame the points in which the horizontal wire of the level will intersect it. A line joining these marks will be the horizontal line. The marks, both for vertical and horizontal, should be made distinct enough on the frame, so as to appear also on the plate after exposure. All errors of adjustment of the camera may be eliminated by making two exposures in different positions of the camera (circle right—circle left) and taking the means of the measurements.

Determination of the Image Distance.—The distance of an image from the common centre of an objective is very near the same as the focal distance. It is of great importance to know this distance accurately. To find it, put up a number of signals and measure carefully the angles between them. From the same point take a photograph of the signals, and after developing the plate measure the horizontal co-ordinates. In the following figure let O be the station, P,A,B,C the signals, α', α'', α''' the angles

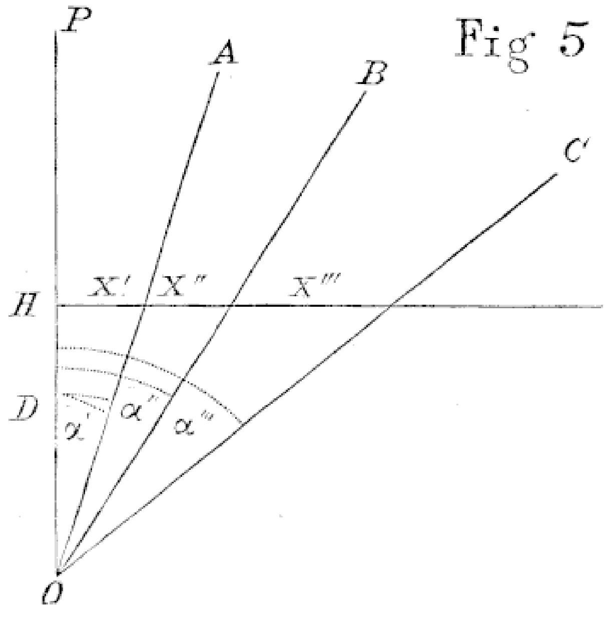


Fig 5

between them. Set up at O and bisect signal P by the vertical line of the camera. Expose a plate, and afterwards measure the co-ordinates x', x'', x'''. Then the distance of the image from the centre of the objective is found by $D = x' \tan \alpha', = x'' \tan \alpha''$, and so on for every angle. If the lens draws in true perspective throughout its field, the values of D should be always the same. The difference between the values of D will give, therefore, a measure of the accuracy of the lens, and the whole operation is at the same time a test for the lens.

The above adjustments are those of a photo-theodolite. I will give now the operations necessary to use an ordinary camera for photogrammetry.

They have been used in 1874 by Dr. Tordan, now professor of geodesy at the Royal Technical College, Hanover, to make a survey of the Oasis Dachel, in the Lybian Desert, in connection with the expedition of Mr. Rohlfs, of which Dr. Tordan was astronomer.

When exposure is mode, the camera axis should be made horizontal by means of levelling screws and levels. After developing the plate, we assume that the centre of the plate is the point in which the vertical and horizontal lines intersect each other. To determine its true position it is necessary to measure with a theodolite two horizontal and two vertical angles. To find the chief vertical we compute by the well-known problem of Pothenot the three-point problem.

Let be in figure (6).

O the station from which the photographs have been taken.

P′, P″, P‴, three points, the angles between which α and β have been measured.

The points P′, P″, P‴, being on the plate in a straight line a and b will be their relative distances. It is now required to find the position of the chief point A and the image distance A O = D. Having measured a and b on the plate, and α and β in the field, we compute the distance s'' from

$$s'' = \frac{a \sin \delta'}{\sin \alpha} = \frac{b \sin \delta'''}{\sin \beta}$$

$$\text{and } \frac{\sin \delta'''}{\sin \delta'} = \frac{a}{\sin \alpha} \div \frac{b}{\sin \beta} = \tan.\ A$$

$$\frac{\sin \delta' - \sin \delta'''}{\sin \delta' + \sin \delta'''} = \frac{1 - \tan.\ A}{1 + \tan.\ A} = \cot \tan.\ (45^\circ + A)$$

$$\tan.\ \frac{\delta' - \delta'''}{2} = \tan \frac{\delta' + \delta'''}{2} \cot \tan\ (45^\circ + A)$$

as $\delta' + \delta''' = 180^\circ - (\alpha + \beta)$ is we may compute δ' and δ''', using also the formula $\tan \frac{\delta' - \delta'''}{2} = \cot \tan.\ \frac{\alpha + \beta}{2} \cot \tan.\ (45^\circ + A)$

Next we compute the three distances

$$(1) \quad s' = \frac{a}{\sin \alpha} \sin \delta''$$

$$(2) \quad s'' = \frac{a}{\sin \alpha} \sin \delta' = \frac{b}{\sin \beta} \sin \delta'''$$

$$(3) \quad s''' = \frac{b}{\sin \beta} \sin \delta''$$

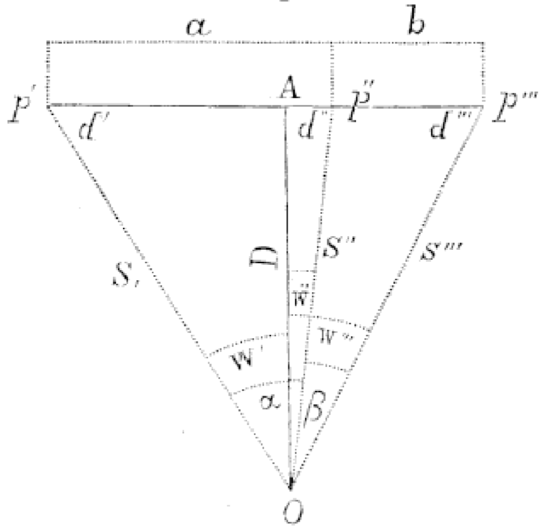
The direction angles W′, W″, W‴, the angles between O A and O P′, O P″, O P‴ are found from

(1) $W' = 90^\circ - \delta'$

(2) $W'' = 90^\circ - \delta''$

(3) $W''' = 90^\circ - \delta'''$

Fig 6



For the image distance D we have therefore three values

$$D = s' \sin. \delta'$$
$$= s'' \sin. \delta''$$
$$= s''' \sin. \delta'''$$

or

$$D = s' \cos W'$$
$$= s'' \cos W''$$
$$= s''' \cos W'''$$

or expressed in terms of a and b

$$D = \frac{a \cos W' \cos W''}{\sin. (W'' - W'} = \frac{b \cos W'' \cos W'''}{\sin. (W''' - W''}$$

We next compute the values x', x'', x''', or the distances AP′, AP″, AP‴, by

$$x' = s' \sin. W' \qquad x'' = s'' \sin. W'' \qquad x''' = s''' \sin. W'''$$
$$= D \tan. W' \qquad = D \tan. W'' \qquad = D \tan. W'''$$

These are the absolute values of the co-ordinates either + or − according as the points are to the right or to the left of A.

To check against any error in the computation we may compute every value by the double formula given. The final check both on computation and measurement is that

$$x'' - x' = a \text{ and } x''' - x'' = b.$$

The true position of A is then found by measuring the length x' from P′ towards the right if P′ is to the left of the centre or x''' from P‴ towards the centre. A may also be found by measuring the co-ordinates of P′, P‴ from the assumed position in the centre of the plate and comparing with the true computed values.

Having formed the chief vertical we next determine the horizontal line, that is the distance of A above or below the centre-line of the plate. Having measured the vertical angles C′ C″ for two points we compute

$$y' = \text{D sec. W}' \; \text{S}' \; \tan C'$$
$$y'' = \text{D sec. W}'' \; \text{S}'' \; \tan C''$$

This, compared with the measured co-ordinates on the plate (from the centre line), will give the amount A has to be moved. It is, of course, only necessary to add these corrections algebraically to the measured co-ordinates. The computations will have to be repeated for every new plate (different view) exposed.

The above formula for finding s may be used as the measurement of a baseline, providing the distance A between the points P′P″ is known, for we have the proportion—

$$\frac{s}{a} = \frac{\text{S}}{\text{A}} \text{ or } \text{S} = \frac{s\text{A}}{a}$$

If we have determined once for all that the point A falls near the centre of the plate, we need use in the future only two points and the horizontal angle between them in order to determine A. We should select the two points such that A will lie between them—for instance, P′P″.

We first measure A = P′P″. Next we compute—

$$\tan. w' = \frac{x'}{\text{D}} \qquad \tan. \text{W}'' = \frac{x''}{\text{D}} \text{ using an (1)}$$

approximate value of D.

Then $\text{W}'' - \text{W}'$ should be $= a$ and $\text{X}'' - \text{X}' = a$.

If this is not the case correct the position of A. Compute D from—

$$\frac{\text{D} = a \cos. \text{W}' \cos. \text{W}''}{\sin. a} \text{ and with D compute } a \text{ (2)}$$

new value of W′ W′ by (1). Repeat the computation till $\text{W}'' - \text{W}''$ and $\text{X}'' - \text{X}''$ will agree with the observed a and the measured a.

Constant Image Distance.—The focal length of a lens is constant above a certain distance of the objects from the lens. For distances below this we may compute the corrections applied to D in order to obtain the true distances.

If f = focal length of lens,

K = distance of object from lens,

then

$$\triangle \text{ D} = f + \frac{f^2}{\text{K}}$$

and for a given value of f we can compute a table given △ D for different values of K (3).

On the Influence of the Errors of Adjustment upon the Results.—With care and usual measuring instruments, such as magnifying glass and a glass scale, a measurement may be made within 1-10th millimetre, or about 1-250th of an inch. This, with a focal distance of 200 millimetres (about 8 inches), gives a value of 1′ – 2′.

Error of the Vertical Position of the Plate.—Suppose the optical axis of the camera to be horizontal and the plate not vertical, but inclined through an angle y, then, if y is the measured co-ordinate, the error $d\,y$ is

$$d\,y = \frac{y^2}{\mathrm{D}} \text{ sin. } y, \text{ or if } y \text{ is small } = \frac{y^2}{\mathrm{D}}\ \frac{y}{3438}$$

This formula is sufficiently accurate for all azimuths.

For the horizontal co-ordinate X we have—

$$d\,\mathrm{X} = \frac{\mathrm{X}\,y}{\mathrm{D}}\ \frac{y}{3438}$$

Error due to the Plate not being at right angles to the Optical Axis of the Camera.—If the plate is not perpendicular to the optical axis, this will intersect the plate not in H, the chief point, but in some other point, whose distance from H will be $\triangle\ x$ then the error of the a is found from

$$d\,a = \frac{\triangle\ x}{\mathrm{D}}\ .\ \frac{x^2}{x^2 + \mathrm{D}^2} = \frac{\triangle\ x}{\mathrm{D}} \text{ sin.}^2\, a$$

Error due to a wrong position of the Horizontal Line.—This error affects the vertical angles only.

$$d\,\beta = \frac{\triangle\ y}{\mathrm{D}} \text{ cos.}^2\, \beta$$

Error due to a Change of the Image Distance.—

For horizontal angles $d\,a = \dfrac{x\ \triangle\ \mathrm{D}}{x^2 + \mathrm{D}^2}$

For vertical angles $d\,\beta = \dfrac{y\ \triangle\ \mathrm{D}}{y^2 + \mathrm{D}^2}$

Error due to the Inclination of the Horizontal Axis.—The inclination of the horizontal axis is measured by the striding level. Let this be i then

$$d\,x = y\frac{i}{3438}$$

$$d\,y = x\frac{i}{3438}$$

Error due to the Shrinkage of the Photograph Film.—With good plates this error is quite inappreciable. Very large errors, however, occur if the measurements are taken from a positive on paper—paper having been known to contract as much as 5 % of its length. It is best, therefore, to use only measurements from glass plates.

In a future paper I will give the application of photography to the determination of latitude and longitude.

MEAN LOCAL CIVIL TIME OF MOON'S TRANSIT OVER THE MERIDAN OF 180° EAST OF GREENWICH.

This table gives the mean civil time of Moon's upper and lower transit, as well as the equation of time.

To save time and make use of the figures given in the Nautical Almanac without alteration, the equation of time is given for mean midnight. This midnight is the midnight at the *beginning* of the civil day whose date it applies to. Thus the equation of time at mean noon at Greenwich on May 5 is the same as for mean midnight at Long 180° E. for May 6; that is the beginning of the civil day, May 6. Also the times of *upper* and *lower* transit given in the table are the same as those in the Nautical Almanac, but the date is increased one day, and "upper" and "lower" transposed. Thus mean astronomical times of *upper* and *lower* transit for May 5, Greenwich, are the mean civil times of *lower* and *upper* transit at 180° E. for May 6. The times are given to the nearest tenth of a minute only, as being quite sufficient for all purposes connected with the tides, and the differences for 1 hour are found by dividing the difference for two successive transits by 12.

The correction for time of moon's transit for any part of New Zealand will always be less than the difference given in the last column, as the whole of New Zealand lies between 165° and 180° E. longitude, but the table will serve equally well for the Australian colonies, by turning the difference of longitude into hours and decimals and multiplying this difference by the number given in the last column. This correction is always additive for places West of longitude 180° East; that is for New Zealand and the Australian colonies, for which the table has been specially prepared.

To avoid the use of "a m." and "p.m." the hours are counted from 0 hours to 24 hours, that is from midnight at the beginning of the civil day to the next midnight.

EXAMPLE I.—Required the apparent civil local time of moon's upper and lower transit at Melbourne on 6th May, 1896 :—

Long. of Melbourne Observatory	= 144° 58′ 27″ E.	
	= 9h. 39·9m. E. in time	
	= 2h. 20·1m. W of 180°	
Moon's upper transit 6th May	= 7h. 6·6m.	Lower = 19h. 27·2m.
Correction 1·7 × 2·33	= + 4·0m.	= + 4·0m.
Equation of time	= + 3·5m.	= + 3·5m.
Melbourne apparent civil time 6th May	7h. 14·1m.	19h. 34·7m.

For places East of 180° E. long. take out the quantities for the day following in the tables, and subtract the correction for the difference of longitude.

EXAMPLE II.—Required the apparent local civil time of moon's upper and lower transits at Apia, Samoa, on 6th May, 1896 :—

Long. of Apia	= 171° 44′ 00″ W	
	= 11h. 26·9m. W in time	
	= 0h. 33·1m. E of 180°	
Moon's upper transit 7th May	= 7h. 47·2m.	Lower = 20h. 6·9m.
Correction 1·7 × ·55	= − 0·9m.	= − 0·9m.
Equation of time	= + 3·6m.	= + 3·6m.
Apia apparent civil time 6th May	7h. 49·9m.	20h. 9·6m.

1896	Equa. of Time.	Moon's Transit. Upper.	Moon's Transit. Lower.	Diff. for 1 hour.	1896	Equa. of Time.	Moon's Transit. Upper.	Moon's Transit. Lower.	Diff. for 1 hour.
Jan	m	h m	h m	m	Mar	m	h m	h m	m
1	— 3·2	— —	12 13·8	2·5	1	— 12·6	0 47·2	13 13·0	2·2
2	— 3·7	0 44·2	13 13·9	2·5	2	— 12·4	1 38·7	14 4·6	2·2
3	— 4·1	1 42·8	14 10·8	2·3	3	— 12·2	2 30·8	14 57·3	2·2
4	— 4·6	2 38·0	15 4·2	2·2	4	— 12·0	3 24·3	15 51·9	2·3
5	— 5·0	3 29·7	15 54·5	2·1	5	— 11·7	4 20·0	16 48·6	2·4
6	— 5·5	4 18·8	16 42·9	2·0	6	— 11·5	5 17·5	17 46·6	2·4
7	— 5·9	5 6·8	17 30·8	2·0	7	— 11·3	6 15·7	18 44·4	2·4
8	— 6·4	5 55·0	18 19·6	2·1	8	— 11·0	7 12·7	19 40·4	2·3
9	— 6·8	6 44·7	19 10·4	2·1	9	— 10·8	8 7.2	20 33·0	2·1
10	— 7·2	7 36·8	20 3·9	2·2	10	— 10·5	8 57·9	21 21·8	2·0
11	— 7·6	8 31·7	21 0·1	2·3	11	— 10·3	9 44·9	22 7·0	1·8
12	— 8·0	9 28·8	21 57·7	2·3	12	— 10·0	10 28·4	22 49·2	1·7
13	— 8·4	10 26·5	22 55·0	2·3	13	— 9·7	11 9·5	23 29·3	1·7
14	— 8·8	11 22·8	23 49·8	2·2	14	— 9·4	11 48·9	— —	1·6
15	— 9·2	12 15·9	— —	2·1	15	— 9.2	12 27·7	0 08·3	1·6
16	— 9·6	13 5·1	0 41·0	2·0	16	— 8·9	13 6·9	0 47·2	1·6
17	— 9·9	13 50·5	1 28·3	1·9	17	— 8·6	13 47·5	1 27·0	1·7
18	— 10·2	14 32·7	2 11·9	1·8	18	— 8·3	14 30·4	2 8·6	1·8
19	— 10·6	15 12·7	2 52·9	1·7	19	— 8·0	15 16·4	2 52·9	1·9
20	— 10·9	15 51·6	3 32·2	1·8	20	— 7·7	16 5·9	3 40·7	2·1
21	— 11·2	16 30·5	4 11·0	1·6	21	— 7·4	16 59·1	4 32·1	2·2
22	— 11·5	17 10·7	4 50·4	1·8	22	— 7·1	17 55·0	5 26·8	2·3
23	— 11·7	17 53·2	5 31·6	1·8	23	— 6·8	18 52·3	6 23·6	2·4
24	— 12·0	18 39·4	6 15·8	1·9	24	— 6·5	19 49·3	7 21·0	2·4
25	— 12·3	19 29·9	7 4·0	2·2	25	— 6·2	20 44·9	8 17·3	2·2
26	— 12·5	20 25·1	7 56·9	2·3	26	— 5·9	21 38·5	9 11·9	2·2
27	— 12·7	21 24·1	8 54·2	2·5	27	— 5·6	22 30·7	10 4·7	2·2
28	— 12·9	22 25·2	9 54·6	2·5	28	— 5·3	23 22·3	10 56·5	2·1
29	— 13·1	23 25·7	10 55·7	2·5	29	— 5·0	— —	11 48·2	2·2
30	— 13·3	— —	11 55·1	2·5	30	— 4·6	0 14·5	12 41·2	2·2
31	— 13·5	0 23·8	12 51·6	2·3	31	— 4·3	1 8·5	13 36·5	2·3
Feb					Apr 1	— 4·0	2 5·1	14 34·3	2·4
1	— 13·6	1 18·6	13 44·9	2·2	2	— 3·7	3 4·1	15 34·2	2·5
2	— 13·8	2 10·6	14 35·9	2·1	3	— 3·4	4 4·4	16 34·4	2·5
3	— 13·9	3 0·8	15 25·6	2·1	4	— 3·1	5 4·0	17 33·0	2·4
4	— 14·0	3 50·4	16 15·5	2·1	5	— 2·8	6 1·0	18 28·0	2·3
5	— 14·1	4 40·9	17 6·7	2.2	6	— 2·5	6 53·9	19 18·8	2·1
6	— 14·2	5 33·1	18 0·1	2.2	7	— 2·3	7 42·5	20 5·3	1·9
7	— 14·3	6 27·6	18 55·6	2·3	8	— 2·0	8 27·2	20 48·4	1·8
8	— 14·3	7 24·0	19 52·5	2·4	9	— 1·7	9 8·9	21 29·0	1·7
9	— 14·4	8 21·0	20 49·3	2·4	10	— 1·4	9 48·7	22 8·1	1·6
10	— 14·4	9 17·1	21 44·2	2·3	11	— 1·1	10 27·5	22 46·9	1·6
11	— 14·4	10 10·6	22 36·0	2·1	12	— 0·9	11 6·5	23 26·4	1·7
12	— 14·5	11 0·5	23 24·0	2·0	13	— 0·6	11 46·7	— —	1·7
13	— 14·4	11 46·7	— —	1·8	14	— 0·4	12 29·0	0 7·6	1·8
14	— 14·4	12 29·7	0 8·5	1·8	15	— 0·1	13 14·2	0 51·2	1·8
15	— 14·4	13 10·3	0 50·2	1·7	16	+ 0·1	14 2·8	1 38·1	2·1
16	— 14.3	13 49·5	1 30·0	1·6	17	+ 0·4	14 54·8	2 28·4	2·2
17	— 14·3	14 28·3	2 08·9	1·6	18	+ 0·6	15 49·5	3 21·9	2·3
18	— 14·2	15 7·8	2 47·9	1·7	19	+ 0·8	16 45·5	4 17·4	2·3
19	— 14·1	15 49·0	3 28·1	1·8	20	+ 1·0	17 41·2	5 13·5	2·3
20	— 14·0	16 32·9	4 10·5	1·8	21	+ 1·3	18 35·5	6 8·6	2·2
21	— 13·9	17 20·5	4 56·2	2·1	22	+ 1·5	19 27·8	7 1·9	2·2
22	— 13·9	18 12·3	5 45·9	2·2	23	+ 1·7	20 18·6	7 53·3	2·1
23	— 13·7	19 8·1	6 39·7	2·4	24	+ 1·9	21 8·6	8 43·6	2·1
24	— 13·6	20 6·6	7 37·1	2·5	25	+ 2·0	21 59·1	9 33·7	2·1
25	— 13·5	21 6·2	8 36·4	2·5	26	+ 2·2	22 51·3	10 24·9	2·2
26	— 13·3	22 4·7	9 35·7	2·4	27	+ 2·4	23 46·5	11 18·5	2·3
27	— 13·1	23 1·1	10 33·3	2·3	28	+ 2·5	— —	12 15·4	2·4
28	— 13·0	23 55·1	11 28·4	2·2	29	+ 2·7	0 45·1	13 15·4	2·5
29	— 12·8	— —	12 21·3	2·3	30	+ 2·8	1 46·3	14 17·4	2·5

1896	Equa. of Time.	Moon's Transit. Upper.	Moon's Transit. Lower.	Diff. for 1 hour.	1896	Equa. of Time.	Moon's Transit. Upper.	Moon's Transit. Lower.	Diff. for 1 hour.
May	m	h m	h m	m	July	m	h m	h m	m
1	+ 3.0	2 48.3	15 18.8	2.5	1	— 3·5	4 18 9	16 38·8	1·7
2	+ 3.1	3 48.6	16 !7.4	2.4	2	— 3·7	4 58·5	17 18·2	1·6
3	+ 3.2	4 45.0	17 11.4	2.2	3	— 3·9	5 37·9	17 57·7	1·6
4	+ 3.3	5 36.5	18 0.5	2.0	4	— 4·0	6 18·0	18 38·7	1·7
5	+ 3.4	6 23.4	18 45.4	1.8	5	- 4·2	7 0·1	19 22·2	1·8
6	+ 3.5	7 6.6	19 27.2	1.7	6	— 4·4	7 45·1	20 9·1	2·0
7	+ 3.6	7 47.2	20 6.9	1.6	7	— 4·6	8 34·0	20 59·9	2·2
8	+ 3.6	8 26.4	20 45.8	1.6	8	— 4·7	9 26·7	21 54·4	2·3
9	+ 3.7	9 5.3	21 25.0	1.6	9	— 4·9	10 22·7	22 51·5	2·4
10	+ 3.7	9 45.0	22 5.5	1.7	10	— 5·0	11 20·4	23 49·3	2·4
11	+ 3.8	10 26.7	22 48.5	1.8	11	— 5·2	12 17·8	— —	2·5
12	+ 3.8	11 11.1	23 34.6	2.0	12	— 5·3	13 13·3	0 45·9	2·3
13	+ 3.8	11 59.0	— —	2.0	13	— 5·4	14 6·0	1 40.0	2·2
14	+ 3.8	12 50.6	0 24.4	2.2	14	— 5·6	14 56·2	2 31·4	2·1
15	+ 3.8	13 45.0	1 17.5	2.3	15	— 5·7	15 44·8	3 20·6	2·0
16	+ 3.8	14 41.1	2 13.0	2.3	16	— 5·8	16 33·0	4 8·8	2·0
17	+ 3.8	15 37.1	3 9.2	2.3	17	— 5·9	17 22·0	4 57·3	2·1
18	+ 3.8	16 31.4	4 4.5	2.2	18	— 5·9	18 13·3	5 47·3	2·2
19	+ 3.7	17 23.4	4 57.7	2.1	19	— 6·0	19 7·6	6 40·0	2·3
20	+ 3.7	18 13.2	5 48.5	2.1	20	— 6·1	20 5.0	7 35·9	2·4
21	+ 3.6	19 1.7	6 37.6	2.0	21	— 6·1	21 4·7	8 34·7	2·5
22	+ 3.6	19 50.1	7 25.9	2.0	22	— 6·2	22 4·5	9 34·7	2·5
23	+ 3.5	20 39.8	8 14.7	2.1	23	— 6·2	23 2·3	10 33·8	2·4
24	+ 3.4	21 32.0	9 5.5	2.2	24	— 6·3	23 56·2	11 29·8	2.2
25	+ 3.3	22 27.8	9 59.4	2 4	25	— 6·3	— —	12 21·6	2·1
26	+ 3.2	23 27.3	10 57.1	2.5	26	— 6·3	0 45·8	13 9·1	1·9
27	+ 3.1	— —	11 58.1	2.6	27	— 6·3	1 31·5	13 53·0	1·8
28	+ 3.0	0 29.3	13 0.5	2.6	28	— 6·3	2 13·9	14 34·4	1·7
29	+ 2 9	1 31.4	14 1.6	2.5	29	— 6·2	2 54·4	15 14·2	1·6
30	+ 2.8	2 30.9	14 59.0	2.3	30	— 6·2	3 33·9	15 53·7	1·6
31	+ 2.6	3 25.8	15 51.4	2.1	31	— 6·2	4 13·6	16 33·9	1·7
					Aug.				
June					1	— 6.1	4 54.7	17 16.0	1.8
1	+ 2.5	4 15.8	16 39.1	1.9	2	— 6.1	5 38.1	18 1.0	1.9
2	+ 2.3	5 1.3	17 22.7	1.8	3	— 6.0	6 24.7	18 49.4	2.1
3	+ 2.2	5 43.5	18 3.6	1.7	4	— 5.9	7 15.1	19 41.7	2.2
4	+ 2.0	6 23.4	18 43.0	1.6	5	— 5.8	8 9.2	20 37.3	2.3
5	+ 1.8	7 2.5	19 22.1	1.6	6	— 5.7	9 5.9	21 34.7	2.4
6	+ 1.7	7 41.9	20 2.1	1.7	7	— 5.6	10 3.5	22 32.2	2.4
7	+ 1.5	8 22.7	20 44.9	1.8	8	— 5.5	11 0.4	23 28.1	2.3
8	+ 1.3	9 6.1	21 29.0	1.9	9	— 5.4	11 55.2	— —	2.2
9	+ 1.1	9 52.9	22 17.7	2.1	10	— 5.2	12 47.6	0 21.7	2.1
10	+ 0.9	10 43.5	23 10.1	2.2	11	— 5.1	13 38.2	1 13.1	2.1
11	+ 0.7	11 37.6	— —	2.3	12	— 4.9	14 27.8	2 3.0	2.0
12	+ 0.5	12 34.2	0 5.7	2.4	13	— 4.8	15 17.8	2 52.7	2.1
13	+ 0.3	13 31.2	1 2.8	2.4	14	— 4.6	16 9.4	3 43.3	2.2
14	+ 0.1	14 27.0	1 59.4	2.3	15	— 4.4	17 3.4	4 36.0	2.3
15	— 0.1	15 20.3	2 54.0	2.2	16	— 4.2	18 0.1	5 31.4	2.4
16	— 0.3	16 10.9	3 45.9	2.1	17	— 4.0	18 58.9	6 29.3	2.3
17	— 0.5	16 59.4	4 35.4	2.0	18	— 3.8	19 58.2	7 28.6	2.5
18	— 0.7	17 47.0	5 23.3	2.0	19	— 3.6	20 55.8	8 27.3	2.4
19	— 1.0	18 35.0	6 10.9	2.0	20	— 3.3	21 50.2	9 23.5	2.2
20	— 1.2	19 24.8	6 59.6	2·1	21	— 3.1	22 40.6	10 15.9	2.1
21	— 1.4	20 17.6	7 50.8	2.2	22	— 2.8	23 27.0	11 4.3	1.9
22	— 1.6	21 14.0	8 45.3	2.4	23	— 2.6	— —	11 49·0	1.8
23	— 1.8	22 13.8	9 43.6	2.5	24	— 2.3	0 10.3	12 31.0	1.7
24	— 2.1	23 15.2	10 44.5	2.6	25	— 2.1	0 51.3	13 11.3	1.7
25	— 2.3	— —	11 45.8	2.6	26	— 1.8	1 31.1	13 50.8	1.6
26	— 2.5	0 15.7	12 44.9	2.4	27	— 1.5	2 10.6	14 30.7	1.7
27	— 2.7	1 13.0	13 39.9	2.2	28	— 1.2	2 51.1	15 11.9	1.7
28	— 2.9	2 5.6	14 30.1	2.0	29	— 0.9	3 33.3	15 55.4	1.8
29	— 3.1	2 53.6	15 16.0	1.9	30	— 0.6	4 18.2	16 41.8	2.0
30	— 3.3	3 37.6	15 58.5	1.8	31	— 0.3	5 6.3	17 31.7	2.1

1896	Equa. of Time.	Moon's Transit.		Diff. for 1 hour.
		Upper.	Lower.	
Sept	m	h m	h m	m
1	+ 0·0	5 57·9	18 24·8	2·2
2	+ 0·3	6 52·3	19 20·2	2·3
3	+ 0·6	7 48·4	20 16·7	2·4
4	+ 1·0	8 44·8	21 12·6	2·3
5	+ 1·3	9 40·0	22 7·0	2·2
6	+ 1·6	10 33·5	22 59·6	2·1
7	+ 1·9	11 25·4	23 50·9	2·1
8	+ 2·3	12 16·4	— —	2·1
9	+ 2·6	13 7·6	0 41·9	2·1
10	+ 3·0	14 0·3	1 33·7	2·2
11	+ 3·3	14 55·2	2 27·4	2·3
12	+ 3·7	15 52·8	3 23·7	2·4
13	+ 4·0	16 52·5	4 22·5	2·5
14	+ 4·4	17 52·7	5 22·6	2·5
15	+ 4·7	18 51·3	6 22·3	2·4
16	+ 5·1	19 46·6	7 19·4	2·3
17	+ 5·4	20 37·8	8 12·7	2·1
18	+ 5·8	21 24·9	9 1·8	1·9
19	+ 6·1	22 8·6	9 47·1	1·8
20	+ 6·5	22 49·9	10 29·5	1·7
21	+ 6·8	23 29·8	11 10·0	1·6
22	+ 7·2	— —	11 49·5	1·6
23	+ 7·5	0 9·3	12 29·2	1·7
24	+ 7·9	0 49·3	13 9·8	1·7
25	+ 8·2	1 30·9	13 52·5	1·8
26	+ 8·6	2 14·7	14 37·6	1·9
27	+ 8·9	3 1·3	15 25·8	2·0
28	+ 9·2	3 51 0	16 16·9	2·2
29	+ 9·6	4 43·4	17 10·3	2·2
30	+ 9·9	5 37·6	18 4·9	2·3
Oct				
1	+ 10·2	6 32·2	18 59·4	2·3
2	+ 10·5	7 26·3	19 52·8	2·2
3	+ 10·9	8 19·0	20 44·9	2·1
4	+ 11·2	9 10·5	21 35·8	2-1
5	+ 11·5	10 1.2	22 26·6	2·1
6	+ 11·8	10 52·2	23 18·2	2·2
7	+ 12·1	11 44·7	— —	2·2
8	+ 12·3	12 39·8	0 11·9	2·3
9	+ 12·6	13 38·2	1 8·6	2·5
10	+ 12·9	14 39·3	2 8·5	2·6
11	+ 13·1	15 41·8	3 10·5	2·6
12	+ 13·4	16 43·1	4 12·7	2·5
13	+ 13·6	17 41·0	5 12·6	2·4
14	+ 13·9	18 34·3	6 8·2	2·2
15	+ 14·1	19 22·9	6 59·2	2·0
16	+ 14·3	20 7·6	7 45·7	1·8
17	+ 14·5	20 49·4	8 28·8	1·7
18	+ 14·7	21 29·5	9 9·6	1·7
19	+ 14·9	22 8·8	9 49·2	1·6
20	+ 15·1	22 48·5	10 28·6	1·7
21	+ 15·3	23 29·6	11 8·8	1·7
22	+ 15·4	— —	11 50·8	1·8
23	+ 15·6	0 12·7	12 35·3	1·9
24	+ 15·7	0 58·6	13 22·7	2·0
25	+ 15·8	1 47·4	14 12·8	2·1
26	+ 15·9	2 38·8	15 5·2	2·2
27	+ 16·0	3 31·9	15 58·7	2·2
28	+ 16·1	4 25·5	16 52·1	2·2
29	+ 16·2	5 18·3	17 44·3	2·1
30	+ 16·2	6 9·8	18 34·9	2·1
31	+ 16·3	6 59·8	19 24·3	2·0

1896	Equa. of Time.	Moon's Transit.		Diff. for 1 hour.
		Upper.	Lower.	
Nov.	m	h m	h m	m
1	+ 16.3	7 48.8	20 13.3	2.0
2	+ 16.3	8 37.9	21 2.8	2.1
3	+ 16.3	9 28.2	21 54.3	2.2
4	+ 16.3	10 21.3	22 49.1	2.3
5	+ 16.3	11 18.0	23 47.9	2.5
6	+ 16.3	12 18.7	— —	2.6
7	+ 16.2	13 22.4	0 50.3	2.7
8	+ 16.2	14 26.7	1 54.7	2.7
9	+ 16.1	15 28.5	2 58.1	2.5
10	+ 16.0	16 25.6	3 57.7	2.3
11	+ 15.9	17 17.4	4 52.2	2.1
12	+ 15.8	18 4.4	5 41.5	1.9
13	+ 15.6	18 47.6	6 26.4	1.8
14	+ 15.5	19 28.4	7 8.3	1.7
15	+ 15.3	20 8.0	7 48.3	1.6
16	+ 15.2	20 47.4	8 27.6	1.6
17	+ 15.0	21 28.0	9 7.5	1.7
18	+ 14.8	22 10.4	9 48.9	1.8
19	+ 14.5	22 55.6	10 32.6	1.9
20	+ 14.3	23 43.8	11 19.3	2.0
21	+ 14.1	— —	12 9.0	2.0
22	+ 13.8	0 34.8	13 1.2	2.2
23	+ 13.5	1 27.9	13 54.8	2.2
24	+ 13.3	2 21.7	14 48.3	2.2
25	+ 13.0	3 14.7	15 40.6	2.2
26	+ 12.7	4 6.0	16 30.9	2.1
27	+ 12.3	4 55.4	17 19.4	2.0
28	+ 12.0	5 43.2	18 6.8	1.9
29	+ 11.7	6 30.3	18 54.0	1.9
30	+ 11.3	7 18.0	19 42.5	2.0
Dec				
1	+ 10·9	8 7·7	20 33·7	2·1
2	+ 10·6	9 0·8	21 28·9	2·3
3	+ 10·2	9 58·1	22 28·5	2·5
4	+ 9·8	10 59·8	23 31·9	2·7
5	+ 9·3	12 4·3	— —	2·7
6	+ 8·9	13 8·6	0 36 7	2·7
7	+ 8·5	14 9·7	1 39·7	2·5
8	+ 8·1	15 5·6	2 38·3	2·3
9	+ 7·6	15 56·1	3 31·5	2·1
10	+ 7·2	16 42·0	4 19·6	1·9
11	+ 6·7	17 24·5	5 3·6	1·7
12	+ 6·2	18 5·0	5 45·0	1·7
13	+ 5·8	18 44·8	6 24·9	1·7
14	+ 5·3	19 24·9	7 4·7	1·7
15	+ 4·8	20 6·6	7 45·5	1·8
16	+ 4·3	20 50·7	8 28·3	1·9
17	+ 3·9	21 37·9	9 13·9	2·0
18	+ 3·3	22 28·3	10 2·7	2·1
19	+ 2·9	23 21·3	10 54·6	2·2
20	+ 2·4	— —	11 48·4	2·3
21	+ 1·9	0 15·7	12 42·9	2·3
22	+ 1·4	1 9·9	13 36·5	2·2
23	+ 0·9	2 2·6	14 28·1	2·1
24	+ 0·4	2 53·0	15 17·4	2·0
25	— 0·1	3 41·4	16 4·9	2·0
26	— 0·6	4 28·3	16 51·5	1·9
27	— 1·1	5 14·8	17 38·4	2·0
28	— 1·6	6 2·4	18 27·0	2·1
29	— 2·1	6 52·3	19 18.6	2·2
30	— 2·6	7 45·9	20 14·3	2·4
31	— 3·1	8 43·7	21 14·1	2·5
32	— 3·5	9 45·3	22 17·0	2·6

SOLUTION OF PROBLEM.

The problem given by Mr. Halligan on page 174 of the New South Wales "Surveyor" of August, 1895, turns out to be more difficult than it appears at first sight. It is also a very interesting problem, and I propose to make a few remarks on it.

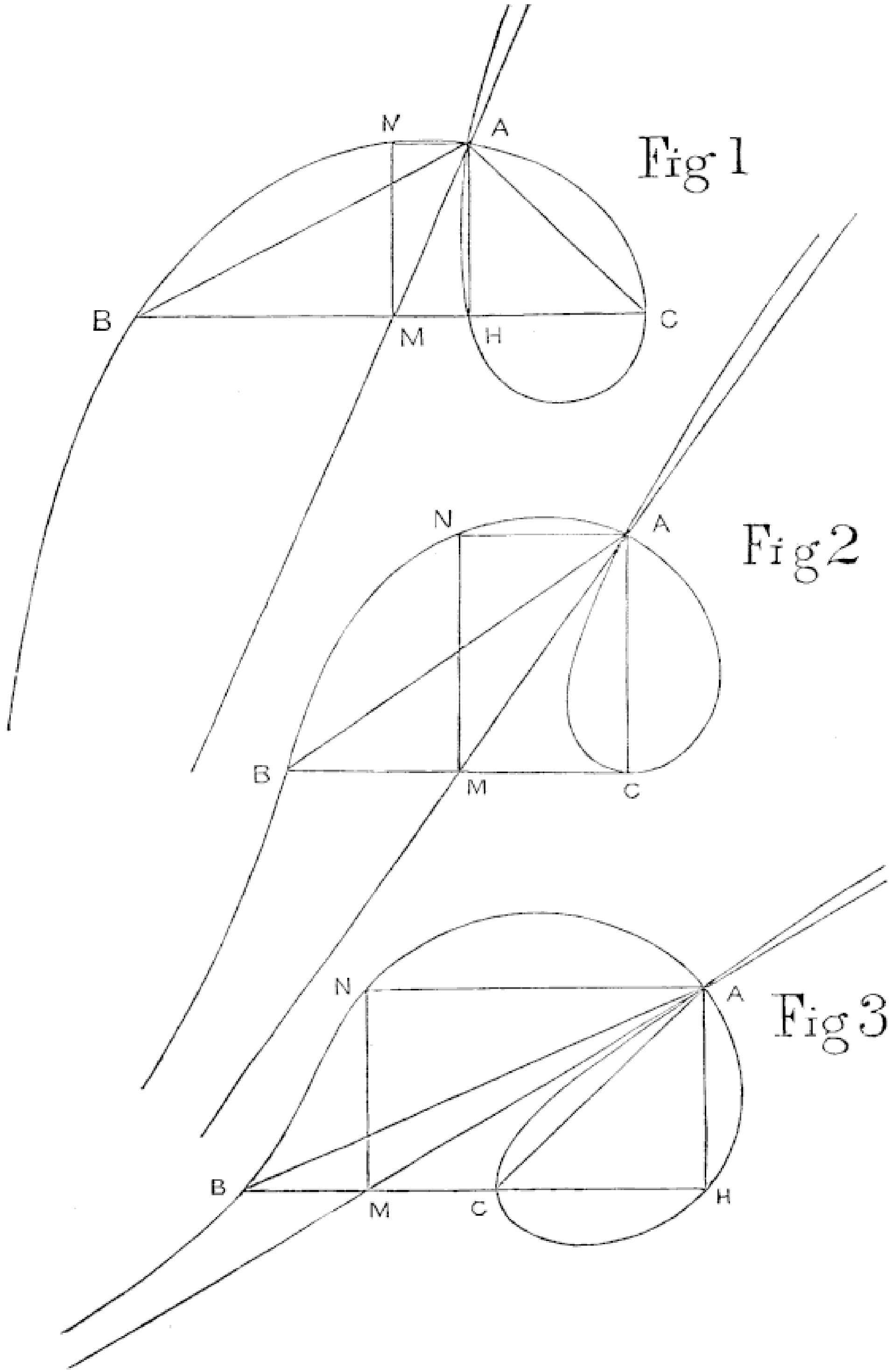
The problem is: "Given the three sides of a triangle ABC, and the length of the line AD to a point D within the triangle, and such that the angles ADB, ADC may be equal. Required the length of the lines DB and DC."

In our last issue I gave a solution of this problem by Mr. A. Beverly, and he has now sent me three diagrams (see figs. 1, 2 and 3) and a note on some of the properties of the curve showing the locus of the point D. From the diagrams it will be seen that there are three cases, or variations of the curve, according as the perp. from A on the base falls within or without the triangle, or on one side thereof. In the diagrams M is the middle point of the base, and a line through AM is an asymptote to both branches of the curve. A perp. from M meeting the curve is equal to the perp. from A to the base.

The tangents to the curve at A are at right angles to each other, bisecting angle A and its externals.

The maximum distance of the curve from AM is the same on both sides.

It will be observed that that part of the curve which is within the triangle lies wholly between the bisector of angle A and the perp. from A. to the base, or, in cases where the perp. falls outside the triangle, then between the bisector of angle A and the shorter side of the triangle.



The simplest way to find the locus of the point D is to draw any number of concentric circles, with A as centre, and radius not exceeding the shortest of the two sides AB and AC.

Then lines drawn from B and C, touching any circle, will intersect at the points D, E, F, G (see fig. 4), and lines drawn from A to each of these points will bisect the angles at D and E, subtended by the base, and will bisect the supplements of the angles at F and G, subtended by the base.

If the radius of the tangent circle shown in Fig. 4 were given, the

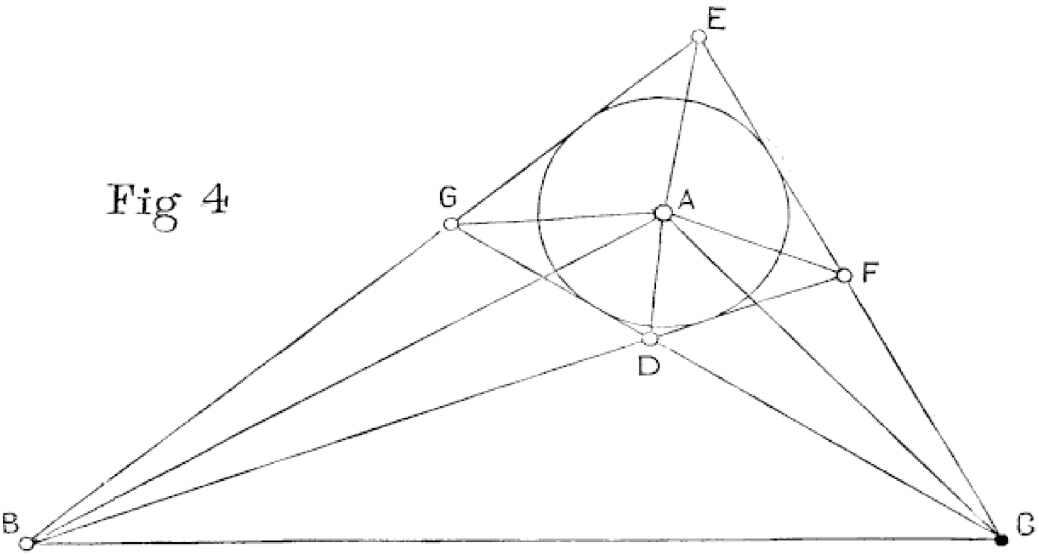


Fig 4

problem would be quite simple, and the distances A D, A E, A F and A G, and the other lines and angles in the figure could easily be calculated. But in the problem as stated by Mr. Halligan, the distance A D is a given length, and Fig. 5 represents the four cases, with the points D, E, F and G at the

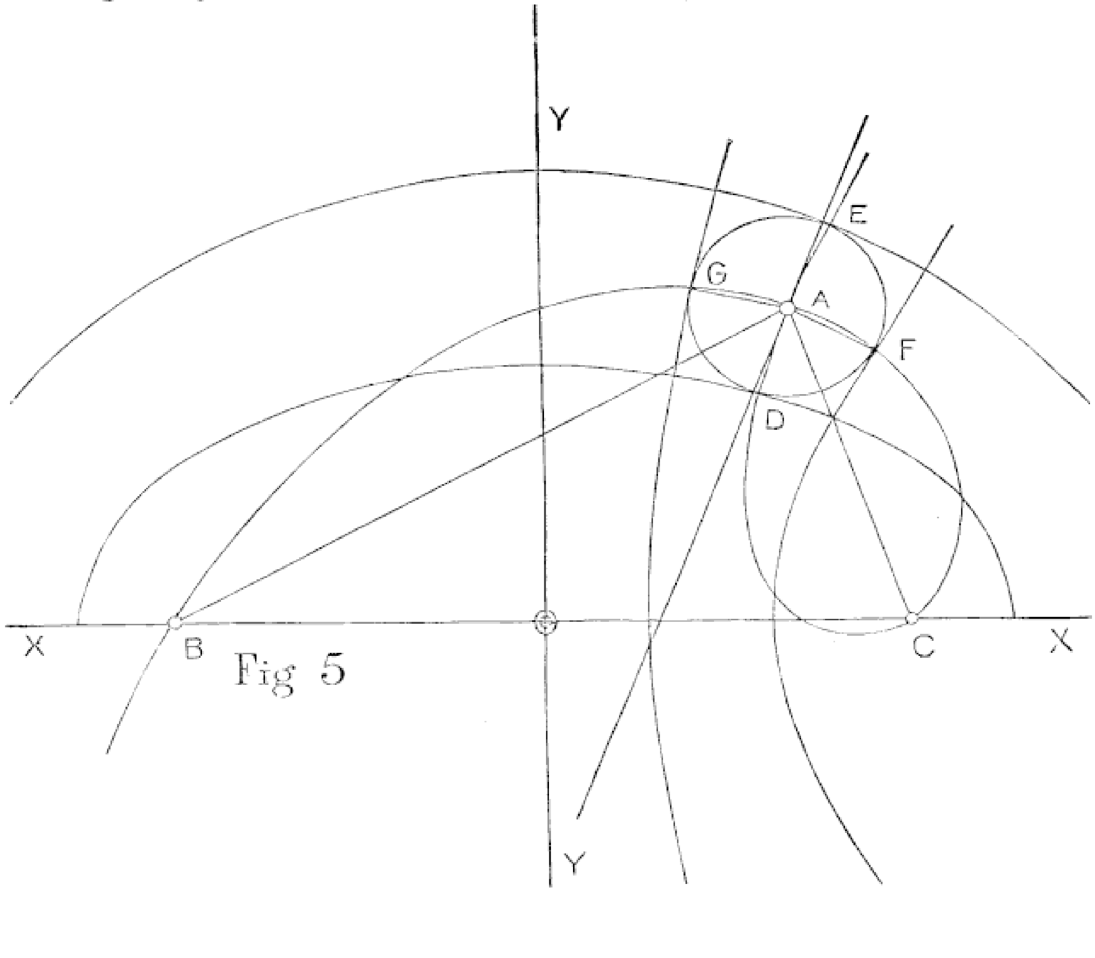


Fig 5

same distance from A, and therefore all on the circumference of a circle. Supposing this circle to represent a cylindrical mirror, then a ray of light from B would be reflectd from the exterior surface of the mirror at D, or the interior surface at E to an observer at C.

Again, supposing B and C to be luminous points, an observer on a line B F produced would see C reflected externally at F, while if on C F produced, he would see B reflected internally at F.

Similarly $\begin{matrix}\text{B}\\\text{C}\end{matrix}$ would be reflected $\begin{matrix}\text{externally}\\\text{internally}\end{matrix}$ at G to an observer on the line $\begin{matrix}\text{C G}\\\text{B G}\end{matrix}$ produced.

Again, taking B and C as foci, then D and E are points of contact of ellipses with the given circle, and F and G points of contact of hyperbolas with the given circle.

Therefore, of all the lines that can be drawn from B and C to the circumference of the circle D F E G, the sum of B D, D C is a minimum, while the sum of B E, E C is a maximum.

Also, of all lines that can be drawn from B and C to the circumference of the circle, the difference of B F and F C is a maximum, and the difference of B G and G C is a minimum, that is when, as in Fig. 5 the hyperbolas are both on the same side of the conjugate axis $m\ n$. If, however, the point A should be nearer the line $m\ n$, or the radius of the circle larger, then if the point of contact G should be to the left of the conjugate axis, then both contacts are external, and the difference is in both cases a maximum.

In our last issue (September, 1895) Mr. Beverly solved the problem by finding an equation giving the value of sin θ, the angle θ being the angle between the line A D and a perp. to the base.

Mr. W. T. Neill of the Survey Department, Dunedin, N.Z., has also obtained a solution, but the angle θ whose sine he obtains is the angle between the line A D and the bisector A H of the angle A (see Fig. 5). Let a, b, c be the sides of the triangle opposite the angles A, B, C ; also let A D = d.

The following is Mr. Neill's investigation :—

Let A B C denote the triangle (Fig. 6). Bisect the angle A by the line A H ; let D, E, F and G be the points of intersection of the curve and a circle described from centre A with radius A D. Join A D, A E, A F and A G. Take $a = 4$, $b = 2$, $c = 3$, and $d = 1$.

Let the angle D A H = θ. A D B = A D C = ϕ

Then D A B = $\frac{A}{2} + \theta$. D A C = $\frac{A}{2} - \theta$.

Then A B D = $\pi - \left\{\phi + \frac{A}{2} + \theta\right\}$ and A C D = $\pi - \left\{\phi - \frac{A}{2} - \theta\right\}$.

From the triangle A B D. $\frac{d}{c} = \frac{\sin\left\{\phi + \frac{A}{2} + \theta\right\}}{\sin\phi}$ (1)

and from triangle A D C. $\frac{d}{b} = \frac{\sin\left\{\phi + \frac{A}{2} - \theta\right\}}{\sin\phi}$ (2)

Expanding (1) $d \sin\phi = c\left\{\sin\phi\cos\left(\frac{A}{2} + \theta\right) + \cos\phi\sin\left(\frac{A}{2} + \theta\right)\right\}$

therefore $d = c\cos\left\{\frac{A}{2} + \theta\right\} + \cot\phi . c\sin\left\{\frac{A}{2} + \theta\right\}$

therefore $\cot\phi = \frac{d - c\cos\left\{\frac{A}{2} + \theta\right\}}{c\sin\left\{\frac{A}{2} + \theta\right\}}$.

Similarly from (2) $\cot\phi = \frac{d - b\cos\left\{\frac{A}{2} - \theta\right\}}{b\sin\left\{\frac{A}{2} - \theta\right\}}$.

Equating $\frac{d - c\cos\left\{\frac{A}{2} + \theta\right\}}{c\sin\left\{\frac{A}{2} + \theta\right\}} = \frac{d - b\cos\left\{\frac{A}{2} - \theta\right\}}{b\sin\left\{\frac{A}{2} - \theta\right\}}$.

therefore $b\,c\sin 2\,\theta + d\,b\sin\left\{\frac{A}{2} - \theta\right\} - d\,c\sin\left\{\frac{A}{2} - \theta\right\} = 0$

and $\sin 2\,\theta - \sin\theta\,\frac{d}{b\,c}(b + c)\cos\frac{A}{2} + \cos\theta\,\frac{d}{b\,c}(b - c)\sin\frac{A}{2} = 0$

Let $m = \frac{d}{b\,c}(b + c)\cos\frac{A}{2}$ and $n = \frac{d}{b\,c}(b - c)\sin\frac{A}{2}$.

Then $\sin 2\,\theta - m\sin\theta + n\cos\theta = 0$ (3)

Square, and remembering that $\cos^2\theta = 1 - \sin^2\theta$ we obtain

$$\sin^2 2\theta + 2n\cos\theta\sin 2\,\theta + n^2\cos^2\theta = m^2\sin^2\theta$$

therefore $\sin^4\theta + n\sin^3\theta + \frac{m^2 + n^2 - 4}{4}\sin^2\theta - n\sin\theta - \frac{n^2}{4} = 0.$ (4)

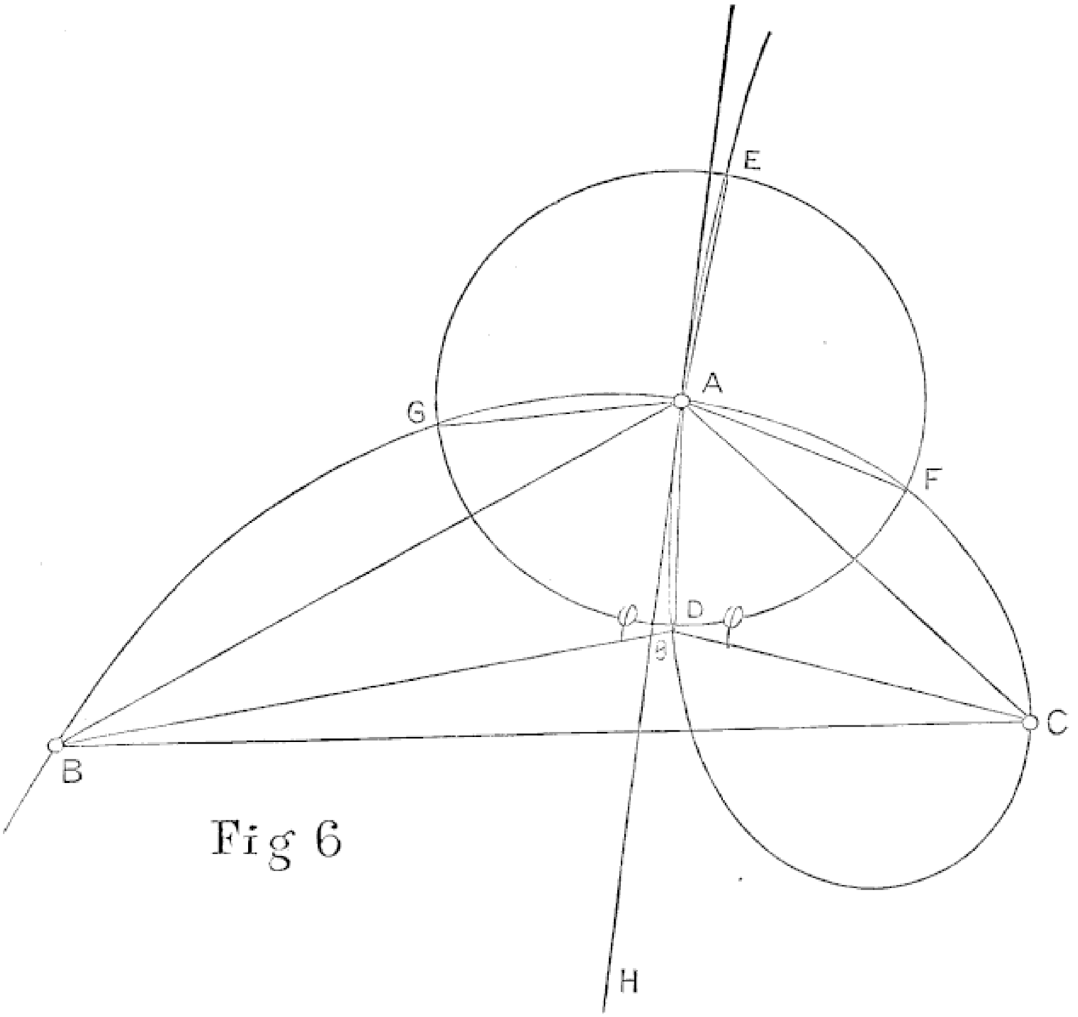


Fig 6

This equation being a bi-quadratic has four roots, and sin θ has therefore four different values (see Fig. 6).

Now the circle cuts the curve at four points, D, E, F and G, and, since any of these positions satisfy the above equations, we see that sin θ has four real values.

We will now solve the equation (4). This can be done by any of the well-known methods, but on account of the co-efficients of the powers of sin θ being fractional, the process is a long and tedious one, so we will resort to an artifice to facilitate the extraction of the roots.

To find the co-efficients :—

$$\cos \text{A} = \frac{b^2 + c^2 - a^2}{2\,b\,c} = \frac{4 + 9 - 16}{2 \,.\, 2 \,.\, 3} = -\frac{1}{4} = -\cdot 25$$

therefore $\cos(\pi - \text{A}) = \cdot 25$

$$\text{A} = \pi - 75° \; 31'\cdot 349 = 104° \; 28'\cdot 651$$

$$\frac{\text{A}}{2} = 52° \; 14'\cdot 325.$$

Now $m = \frac{d}{b\ c}(b + c) \cos \frac{A}{2}$. and $n = \frac{d}{b\ c}(b - c) \sin \frac{A}{2}$.

$= \frac{5}{6\cdot} \cdot 6123726$ $= -\frac{1}{6} \cdot 7905694.$

$= \cdot 5103105$ $= - \cdot 1317615\dot{6}$

$m^2 = \cdot 2604168$ $n^2 = \cdot 0173611105$

$\frac{m^2 + n^2 - 4}{4} = - \cdot 9305555$ $\frac{n^2}{4} = \cdot 0043402776.$

Substituting the numerical co-efficients in (4) we obtain :—

$$\sin^4\theta - \cdot 131761\dot{6} \sin^3\theta - \cdot 930555 \sin^2\theta + \cdot 131761\dot{6} \sin\theta - \cdot 0043403 = 0$$

Observing that the first two terms are small we can neglect them and obtain an approximate value of $\sin\theta$ from a quadratic, thus :—

$$\sin^2\theta - \frac{\cdot 1317616}{\cdot 9305555} \sin\theta + \frac{\cdot 0043403}{9305555} = 0 \qquad \text{(A)}$$

$$\sin^2\theta - \frac{\cdot 1317616}{9305555} \sin\theta + (\cdot 070798)^2 = \cdot 005012 - \cdot 004686$$

$$\sin\theta = \cdot 070798 \pm \cdot 018064$$
$$= 088862 \text{ or } \cdot 052734$$

Therefore approximately $\theta = 5^\circ\ 06'$ or $3^\circ\ 01'$.

To obtain a more accurate value of θ we take equation (3)

$$\sin 2\theta - m \sin\theta + n \cos\theta = 0$$
$$\text{or } 2 \sin\theta = m \tan\theta - n$$

and using two near values of θ, we can get the true value by double position. Thus, if we assume $\theta = 5^\circ\ 04'$, then $\sin\theta$ obtained from $\frac{m \tan\theta - n}{2} =$ $= \cdot 0885032$, and $\theta = 5^\circ\ 04'\cdot 6501$, or $\cdot 6501$ in excess of the assumed value.

If we assume $\theta = 5^\circ\ 05'$ then $\sin\theta$, obtained from $\frac{m \tan\theta - n}{2} =$ $\cdot 0885780$, and $\theta = 5^\circ\ 04'\cdot 9082$, or $\cdot 0918$ in defect of the assumed value.

Then $\frac{6501}{6501 + 918} = \cdot 8763$, therefore $\theta = 5^\circ\ 04'\cdot 8763$ and $\sin\theta =$ $\cdot 0885688$.

Again, by examining the figure we see that the value of θ ($3\cdot 01'$) corresponds to the point E, and in this case $2 \sin\theta = -n - m \tan\theta$, that is, $\cdot 13176156 - \cdot 5103105 \tan\theta$.

assuming $\theta = 3^\circ\ 00'$, the result is $0'\cdot 5947$ in excess

assuming $\theta = 3^\circ\ 01'$, the result is $0\cdot 6618$ in defect

$\frac{5947}{5947 + 6618} = \cdot 4733$ therefore the true value of $\theta = 3^\circ\ 00\cdot 4733$, and $\sin\theta = \cdot 0524735$.

To find the remaining two roots divide equation (4) by ($\sin \theta - \cdot 08856875$). The quotient is

$\sin^3\theta - \cdot 0431928 \sin^2\theta - \cdot 9343810 \sin\theta + \cdot 0490046$. Divide again by ($\sin\theta - \cdot 0524735$) we obtain :—

$$\sin^2\theta + \cdot 0092807 \sin\theta - \cdot 9338940 = 0$$

from which we find $\sin\theta = \pm \cdot 9663931 - \cdot 0046404.$

$$= \cdot 9617525 \text{ or } - \cdot 9710335$$

$$\text{therefore } \theta = 74^\circ.\ 06'.\ 09'' \text{ or } - (76^\circ\ 10'\ 33'')$$

The angle E A H $= 74^\circ\ 06'\ 09''$ and angle G A H $= 76^\circ\ 10'\ 33''$.

Having found θ we can find ϕ from the equation $\cot\phi = \dfrac{d - c\cos\left\{\dfrac{A}{2} + \theta\right\}}{c\sin\left\{\dfrac{A}{2} + \theta\right\}}$.

Then B D and D C can be determined in the usual way by the rule of sines.

When the triangle is isosceles the locus of the point is a straight line through the angle A bisecting the base, and a circle circumscribing the triangle.

We can extract the roots of equation (4) directly by Horner's Method in the following manner :—

sin $^4\theta$	sin $^3\theta$	sin $^2\theta$	sin θ	
1	− ·1317616	− ·9305555	+ ·13176156 −	0043402776 (·0885687
	+ ·08	− 41409	− 74775712 +	45588683
				1 ——
	− ·0517616	− ·9346964	+ 056985854 +	·0002185907
	+ ·08	+ 22591	− 74594984 −	1998951
			1 —— **1**	**2** ——
	+ ·0282384	− ·9324373	− ·017609150	186956
	+ ·08	+ 0086591	− 7377766	164054
		1 —— **1**		**3** ——
	+ ·1082384	− ·9237782	− ·024986896	22902
	+ ·08	+ 15699	− 7364595	19995
	—— **1**		**2** —— **2**	
	·1882384	− ·9222083	− ·032351491	2907
	8	+ 16339	− 459383	2670
	+1962384	− ·9205744	− ·032810874	237
	8	+ 16979	− 459327	231
		2 —— **2**	**3** —— **3**	
	2042384	− ·9188765	− ·033270201	6
	8	+ 1104	− 55	
	2122384	− ·9187661	− ·033325	
	8	+ 1106	− 55	
	—— **2**			
	·2202384	− ·9186555	− ·033580	
	5	+ 1108	− 7	
		3 —— **3**		
	2207384	− ·9185447	− ·033387	
	5	+ 13		
	2212	− ·918531		
	5	+ 13		
	2217	− ·918518		
	5	+ 13		
	—— **3**			
	2222	− ·918505		

Mr. A. Beverly has a process for extracting the roots of higher equations; the following is an example :—

The co-efficients in order are

(x)	1	− ·1317616	− ·9305555	+ ·13176156	− ·004340278	(12)	$\frac{1}{12}$ = ·08333333
(12 x)	1	− 1·58114	− 133·99999	+ 227·68398	− 90·00000		$\frac{1}{12 \cdot 16}$ = 520834
		+ 4	+ 1·25658	− 268·74340	+ 93·10285		
(12 x − 1)	1	+ 2·41886	− 132·74341	− 41·05942	+ 3·10285	(16)	2704
(192 x − 16)	·000244	+ ·00945	− 8·29646	− 41·05942	+ 49·64560		·08856871 = sin θ.
		+ ·00097	+ ·02891	− 16·56360	− 49·34619		
(192 x − 17)		+ ·01042	− 8·26665	− 57·62302	+ ·29941	(192·6)	

$$\left(-\frac{8 \cdot 2666}{193} - 57 \cdot 62302 = -57 \cdot 66585, \text{ and } \frac{57 \cdot 6658}{\cdot 29941} = 192 \cdot 6\right)$$ (This saves finding another factor)

If 193 were used for the third factor, the result would be nearly correct, but not sufficiently so.

The device in parenthesis is easy, and gives very nearly the same result as using an additional factor.

He also obtains the co-efficients of powers of sin θ in a neat manner, thus when, $d = 1\cdot$

$$\cos\frac{A}{2} = \sqrt{\frac{\overline{b+c}^2 - a^2}{4\,b\,c}} = \sqrt{\frac{9}{24}} = \frac{\sqrt{6}}{4} = \cdot 6123724357$$

$$\sin\frac{A}{2} = \sqrt{\frac{a^2 - \overline{b-c}^2}{4\,b\,c}} = \sqrt{\frac{15}{24}} = \frac{\sqrt{10}}{4} = \cdot 7905694150$$

$$m = \frac{c+b}{b\,c}\cos\frac{A}{2} = \frac{5\sqrt{6}}{24} = \cdot 5103103631$$

$$m^2 = \frac{25}{96} = \cdot 260416\dot{6}$$

$$n = \frac{c-b}{b\,c}\sin\frac{A}{2} = -\frac{\sqrt{10}}{24} = -\cdot 1317615692$$

$$n^2 = \frac{5}{288} = \cdot 017361\dot{1}$$

Let sin $\theta = x$ then the equation becomes

$$x^4 - n\,x^3 - \left\{1 - \frac{m^2 + n^2}{4}\right\} x^2 + n\,x - \frac{n^2}{4} = 0$$

or $$x^4 - \frac{\sqrt{10}}{24}x^3 - \frac{67}{72}x^2 + \frac{\sqrt{10}}{24}x - \frac{5}{1152} = 0$$

From which can be found the roots, one value being

$$x = \cdot 0885688487.$$

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